SURFACE FRICTION AT A PERMEABLE PLATE

V. I. Voronin and A. E. Blazhkov

UDC 532.526.2

An approximate solution is proposed for the problem of determining the friction at a permeable plate. Expressions are given for the tangential stresses with continuous uniform injection through a transverse slot.

The influence of injection (evacuation) on surface friction in laminar flow past a plate has been investigated by many authors [1-3] with the aid of integral relationships or numerical methods.

Here we propose a simple approximate relationship in terms of hypergeometric functions.

When $\mu \sim T$, a compressible laminar boundary layer is described in u, ξ variables by the equation

$$Z^2 \frac{\partial^2 Z}{\partial u^2} = -\frac{u}{L} \frac{\partial Z}{\partial \xi}, \text{ where } Z = \frac{\nu_0 \partial u^*/\partial \eta^*}{\sqrt{\nu_0 u_\infty^3}}, \; u = \frac{u^*}{u_\infty}, \; \xi = \frac{\xi^*}{L}. \tag{1}$$

The boundary conditions are $\partial Z/\partial u = V_0^*(\xi)/\sqrt{\nu_0 u_\infty}$ for u = 0, Z = 0 for u = 1, Z = ∞ for $\xi = 0$.

We represent the desired solution in the form $Z = Z_b + Z_1$, where $Z_b = f''/\sqrt{\xi L}$ is the Blasius solution for an impenetrable plate; Z_1 is the solution that depends on injection.

In the linear approximation (Z_1 small), the problem reduces to determination of Z_1 from the equation

$$f''^2 \frac{\partial^2 Z_1}{\partial u^2} - uZ_1 = u\xi \frac{\partial Z_1}{\partial \xi}. \tag{2}$$

It is a fairly complex matter to find an exact solution of (2). We can find an approximate solution, however, replacing f" by a certain approximating function

$$f'' \approx 0.332 \sqrt{1-u^3}. \tag{3}$$

Then making the substitution of variables $t = u^3$, we can write Eq. (2) as

$$t(1-t) \frac{\partial^2 Z_1}{\partial t^2} + \frac{2}{3}(1-t)\frac{\partial Z_1}{\partial t} - Z_1 = \xi \frac{\partial Z_1}{\partial \xi}. \tag{4}$$

Solving by separation of variables, we obtain a system

$$t(1-t)Q''(t) + \frac{2}{3}(1-t)Q'(t) - (n+1)Q(t) = 0, \tag{5}$$

$$\frac{G'(\xi)}{G(\xi)} = \frac{n}{\xi}. \tag{6}$$

The solution of (6) is $G(\xi) = \xi^n$ (n = 0, 1, 2, ...). The hypergeometric equation (5) has the solution [4]

$$Q = AQ_1(t) + BQ_2(t). \tag{7}$$

Here

$$Q_1 = F(\alpha, \beta, \gamma; t) = F\left[\frac{-1 \pm i\sqrt{36(n+1)-1}}{6}, \frac{-1 \mp i\sqrt{36(n+1)-1}}{6}, \frac{2}{3}; t\right],$$

$$Q_2 = t^{1-\gamma}F(\alpha_1, \beta_1, \gamma_1; t) = t^{1-\gamma}F(1-\gamma+\alpha, 1-\gamma+\beta, 2-\gamma; t).$$

Voronezh Polytechnical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 17, No. 2, pp. 342-345, August, 1969. Original article submitted October 2, 1968.

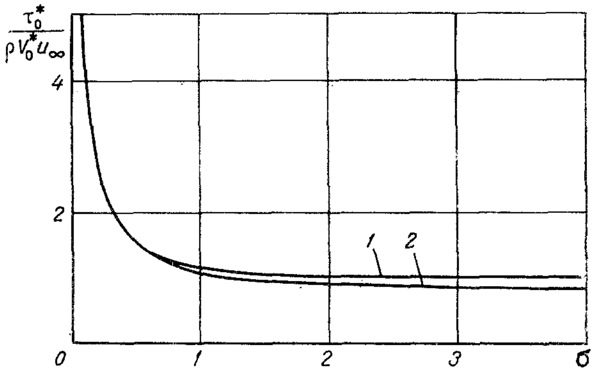

Fig. 1. Comparison of tangential stresses for uniform evacuation: 1) Lew and Fanucci; 2) results of the present study.

Assuming that the solution Z_1 is regular in ξ for u = 0, we find a solution to (4):

$$Z_1 = \sum_{n=0}^{\infty} (A_n Q_{1n} + B_n Q_{2n}) \xi^n. \tag{8}$$

If $V_0^*(\xi)$ is an analytic function, it is easy to determine B_n by expanding the function $(\partial Z_1/\partial u)_{u=0}$ into power series equating the coefficients on identical powers of ξ. The coefficients A_n are found from the conditions on the outer boundary:

$$A_n = -\frac{Q_{2n}(1)}{Q_{1n}(1)} B_n$$

$$= -\frac{\Gamma(\gamma_1)\Gamma(\gamma_1 - \alpha_1 - \beta_1)\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)}{\Gamma(\gamma_1 - \alpha_1)\Gamma(\gamma_1 - \beta_1)\Gamma(\gamma - \alpha - \beta)\Gamma(\gamma)} B_n.$$

For uniform injection, for example (V_0^* = const)

$$B_0 = \frac{V_0^*(\xi)}{\sqrt{\nu_0 u_\infty}},\ B_1 = 0,\ B_2 = 0,\ \ldots;\ A_0 = -0.640\ B_0.$$

As a consequence

$$Z_1 = \frac{V_0^*}{\sqrt{\nu_0 u_\infty}} (Q_{20} - 0.640\ Q_{10}).$$

Going over to the variables x^*, y^*, we find the tangential stress on the plate when (u = 0),

$$\tau_0^* = 0.332\ \rho_0 \sqrt{\frac{\nu_0 u_\infty^3}{x^*}} - 0.640\ \rho_0 u_\infty V_0^*.$$

Comparison with the exact solution [2] shows that up to σ = 2 we have good agreement (see Fig. 1). When the gas is injected through a transverse slot located a distance L from the front of the plate, if we let $\chi = \ln\xi$ in (4), a solution can be obtained by the operational method. In representation space, we obtain Eq. (5), where n must be replaced by the operator p. The coefficients A and B are found from the appropriate boundary conditions. It is very difficult to find the original. It is easy to find the solution $Z_1(0, \chi)$ for the tangential stress at the plate itself (u = 0). In fact, the function

$$Z_1(0, \chi) \leftarrow\!\!\!\!\div Q(0, p) = A = -\frac{\Gamma(\gamma_1)\Gamma(\gamma_1 - \alpha_1 - \beta_1)\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)}{\Gamma(\gamma)\Gamma(\gamma_1 - \alpha_1)\Gamma(\gamma_1 - \beta_1)\Gamma(\gamma - \alpha - \beta)} B$$

$$= -\frac{\Gamma(\gamma_1)\alpha\beta\Gamma^2(\alpha)\Gamma^2(\beta)\sin\pi\alpha\sin\pi\beta}{\pi^2\Gamma(\gamma)} \frac{V_0^*}{\sqrt{\nu_0 u_\infty}} \frac{1}{p} [1 - \exp(-p\chi_L)]$$

is meromorphic with simple poles $\alpha = -n$ or, what is the same thing, $p_n = -(n-1/6)^2 - 35/36$ (α = 0 is not a pole).

Employing the second Heaviside theorem, we determine $Z_1(0, \chi)$ and then, going over to the variables x^*, y^*, the tangential stress

$$\tau_0^* = 0.332\ \rho_0 \sqrt{\frac{\nu_0 u_\infty^3}{x^*}} - 0.65945 \frac{\sqrt{3}}{\pi} V_0^* \rho_0 u_\infty \sum_{n=1}^{\infty} \left[\left(n - \frac{1}{6}\right)^2 - \frac{1}{36}\left(n - \frac{1}{6}\right)\right] \Gamma^2\left(n - \frac{1}{3}\right)$$

$$\times \left[\left(\frac{L+h}{x^*}\right)^{\left(n-\frac{1}{6}\right)^2 + \frac{35}{36}} - \left(\frac{L}{x^*}\right)^{\left(n-\frac{1}{6}\right)^2 + \frac{35}{36}}\right] \frac{1}{\left(n - \frac{1}{6}\right)^2 + \frac{35}{36}}. \tag{9}$$

The series in (9) converges when $x^* > L$; when $x^* > 2L$, the first two terms will be controlling. It is not difficult to generalize the result to the case of several slots.

NOTATION

x^*, y^*	are the transverse and longitudinal coordinates;
ξ^*, η^*	are the Dorodnitsyn variables;
u^*, V^*	are the tangential and Dorodnitsyn-"distorted" normal velocity components;
$f(\xi)$	is the Blasius function;
$\sigma = (v_0^*/U)\sqrt{(2Ux^*T_\infty/C\nu_\infty T^*)}$	is the injection parameter in Lew and Fanucci notation [2];
p	is a Laplace operator;
h	is the width of the slot;
	standard notation is used for the remaining quantities.

Subscripts

0 are the conditions at the wall;
∞ are the conditions in the undisturbed flow;
* are dimensioned quantities.

LITERATURE CITED

1. C. J. Ruger, AIAA J., 5, No. 5, 923-929 (1967).
2. H. Lew and J. Fanucci, Aeron. Sci., 25, No. 5, 589-597 (1955).
3. G. A. Kulonen, Vestnik Leningradskogo Univ., No. 3, 115-119 (1960).
4. N. N. Lebedev, Special Functions and Their Applications [in Russian], Fizmatgiz (1963).